Indian Statistical Institute, Bangalore

B.Math (Hons.) III Year / M.Math II Year, Second Semester

Mid-Sem Examination

Combinatorics and Graph Theory

Time: 3 hours March 02, 2011 Instructor: B.Bagchi

Maximum marks: 100

1. (a) Prove that every tree on at least two vertices has at least two end vertices (vertices of degree 1).

   (b) Prove that any $n-$ vertex tree has exactly $n-1$ edges.

   (c) Prove or disprove : if a graph has n vertices and $n-1$ edges then it is a tree.

   (d) Prove or disprove : if a tree has exactly two end vertices, then it is a path. [$5 \times 4 = 20$]

2. (a) State and prove Moore's inequality.

   (b) Show that any Moore graph is regular. [$8 + 12 = 20$]

3. (a) Show that, up to isomorphism, there is a unique $16-$ vertex graph G such that G induces a copy of the Petersen graph on the non-neighbors of any vertex, and G is K_3- free.

   (b) Compute the spectrum of G.

   (c) Compute the order of the full automorphism group of G.

   [15+5+5 =25]

4. Let $X_1, X_2, \cdots, X_b$ be b $k-$ subsets of a $v-$ set such that $\sharp\,(X_i \cap X_j) = \lambda$ for $i \neq j$. Here $v > k > \lambda$.

   (a) Show that $b \leq v$.

   (b) Give an example of such a family of sets with $b = v = 7, k = 3, \lambda = 1$.

   [$10 + 10 = 20$]

5. If H is a connected graph with chromatic number 2, then show that H has a unique proper colouring in two colors. What is the number of proper $2-$ colorings of H if H has c connected components ? [10+5 =15]